Seminar on MEC728 - Structure Design and Analysis

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INTRODUCTION

* How should an “Advanced Strength of Materials” course be useful for industry? Although the “Basic Strength of Materials” concepts are mandatory for all mechanical and civil engineers, they must use them in combination with some advanced concepts by working in industry for about 5 to 10 years with Codes, Standards and experienced engineers before becoming efficient as structure analysts. An “Advanced Strength of Materials” course should help young engineers to become efficient within about 5 years working as structure analysts in either small or large industries.

* What subjects would help the most for structure analysts? Having acquired several subjects about advanced strength of materials throughout Engineering, Master and PhD programs, and by working more than five years as structure analyst for small and large industries, professor Lê Văn Ngân found that, although any advanced subject could be helpful for some type of industry or for research, some subjects are more useful than others. Among the least useful for structure specialists: General energy methods including Castigliano’s theorem for deflections of complex problems; Stress strain tensors and 3D stress transformation; Extended theories on elasticity, plasticity, plates and shells … More useful for structure specialists: Designing thin plate structures for avoiding general and local buckling; Limit analysis applied to structure design; Understanding why allowable stresses vary from case to case; Welded joint design …

* A proposed undergraduate engineering course for structure design and analysis:

1. **Title of the course**: “Structure Design and Analysis”.

2. **Prerequisite knowledge**: The students must have acquired following knowledge: Basic strength of materials, Finite Element Method.

3. **Alternation between theories and practices**: In each typical week, there are about 3 hours teaching theoretical subjects including 2 to 3 examples to be solved by hand, followed (one or few days later) by 2 hours of “hands on a FEM software” for comparison with hand calculations. The FEM examples must be prepared with enough guidance so that an average student could complete the answers within 90 minutes. This scenario is repeated for about 13 weeks and for about 7 selected subjects.
Chapter 1. Structures subject to buckling (9 h class + 6 h hands on FEM software)

[1 hour]- Buckling of column: Basic Strength of Materials.

Example 1.1: \( D_c = 42.2 \text{ mm}, D_i = 35.1 \text{ mm} \)
\( E = 200000 \text{ MPa}; S_Y = 250 \text{ MPa}. \) Justify why the design (2) allow a load \( P \) about 5 times higher than the design (1).

Slenderness ratio: \( \lambda = \frac{kL}{r} \)
Euler buckling stress: \( S_{cr} = \frac{\pi^2 E}{\lambda^2} \)
Correction to consider plasticity:
If \( S_{cr} \leq \frac{1}{2} S_Y \) \( \Rightarrow S_{cr} = S_{crE} \)
If \( S_{cr} > \frac{1}{2} S_Y \) \( \Rightarrow S_{cr} = S_Y(1 - \frac{1}{4} \frac{S_Y}{S_{crE}}) \)

[2 hours]- Lateral buckling of beams:

Example 1.2: Welded steel beam loaded as shown on left. The beam is supported vertically but without lateral supports.
(a) Determine the safety factor by using approximate design formulas (not show here);
(b) Carry out linear buckling FEM analysis and compare results

Steel: \( E = 200000 \text{ MPa} \)
400x20 mm flanges
10x1000 mm web

FEM results: The beam would laterally buckle if load is multiplied by \( f_{cr} = 0.52 \) at which bending stress is \( \sigma_{cr} = 58.5 \text{ MPa} \) \( \Rightarrow f_{safety} = 0.52 \) (not safe at all).

[6 hours]- Local buckling of web and flanges:

Example 1.3: Same beam as previous example but the beam is laterally supported (lateral buckling eliminated).
(a) Determine the safety factor by using approximate design formulas (not shown here);
(b) Carry out linear buckling FEM analysis and compare results

FEM results: The beam would locally buckle in web near ends if load is multiplied by \( f_{cr} = 1.41 \) at which web shear stress is \( \tau_{cr} = 64 \text{ MPa} \) (safe but not enough).

Example 1.4: Same beam as example 1.3 but the web near ends is reinforced by vertical stiffeners at 1000 mm intervals.
(a) Determine the safety factor by using approximate design formulas (not show here);
(b) Carry out linear buckling FEM analysis and compare results.

FEM results: According to elastic buckling, the beam would theoretically buckle in web at center region if load is multiplied by \( f_{cr} = 4.54 \) at which bending stress is \( \sigma_{crElastic} = 255 \text{ MPa} \); Correction to consider plasticity: \( \sigma_{cr} = 188 \text{ MPa} \) \( \Rightarrow f_{safety} = 188/56 = 3.37 \) (safe).
Chapter 2. Limit Analysis of Structure (9 h class + 6 h hands on FEM software)

[2 hours]- Elastic perfectly plastic idealization & limit analysis of axially loaded bars

Type 1 example: Toutes les tiges ont la même longueur L, la même section A et sont du même matériau élastique parfaitement plastique de contrainte d'écoulement $S_e$ et de module d'élasticité $E = 800 \, S_e$.

1. Déterminez la charge permise $P$ en fonction de $A$, $S_e$ et $f_s$ ($f_s$ = facteur de sécurité).

2. À l'état limite (l'effondrement) de la structure, quelle est la charge $P_L$ ? ; quelle est la tige qui demeure élastique ? ; quelle est la tige qui atteint l'écoulement la dernière?

[4 hours]- Elastic and fully plastic bending moments & limit analysis of compact beams

1st yield bending

$M_e = \frac{1}{12} S_e$  

Fully plastic

$M_L = \frac{A d_0}{2} S_e$  

Shape factor

$k = \frac{M_L}{M_e}$

Type 2 example:

Calculate the shape factor for each of cross section shown on right.

Type 3 example: Calculate the required fully plastic moment ($M_L$) and select the lightest standard S shape steel beam to resist the load shown with a safety factor of 2, knowing that shape factor of S shape is 1.1 and yield stress of steel is $S_e = 270$ MPa.

Answer: $M_L = 30$ kNm; Choice: S 150 x 18.6

[3 hours]- Limit criteria for combined load: One of the mostly used criteria is:

$\left( \frac{M}{M_L/f_s} \right)^2 + \left( \frac{T}{T_L/f_s} \right)^2 + \left( \frac{V}{V_L/f_s} \right)^2 \leq 1$ for bending $M$, torsion $T$ and shear force $V$.

Type 4 example: Using limit analysis, calculate the allowable load $W$ at end B for a safety factor $f_s = 2$ knowing that the tube is made of steel with yield stress $S_e = 270$ MPa.

Answer: $W_{\text{allowed}} = 14340$ N

Engineers who use basic strength of materials would calculate Von-Mises stress, compare with $S_e/2$ and would conclude $W_{\text{allowed}} = 11960$ N : too conservative.
Chapter 3. Allowable stresses for compact regions – Static load (3 h class)

[3 hours] - Compact region = where buckling is practically not possible. Example: thick plates, short columns …

Although the general design criterion is \( \text{Stress due to load} \leq \text{Allowable stress} \), most engineers do not understand why allowable stresses are different from case to case. Since Codes and Standards establish allowable stresses based on limit analysis, it is helpful for engineers to understand the following subject: The general formula for allowable stress in compact regions is \( \text{Allowable stress} = \frac{\text{Yield stress}}{\text{Safety factor}} \) where the “Shape factor” is different for different loads and shapes according to limit analysis. Some allowable stresses are \( (\sigma_p, \tau_p) \) (subscript \( p \) means “permise” in French):

\[
\begin{align*}
\text{3.1.1 Cas particuliers des chargements seuls : } & F_{\text{axiale}}, V_{\text{tangente}}, T_{\text{torse}}, M_{\text{flexion}} \\
\text{Pour } \sigma_{\text{nom}} = F/A : & \quad \sigma_p = \frac{S_e}{fs} \\
\text{Pour } \tau_{\text{nom}} = V/A : & \quad \tau_p = \frac{\tau_e}{fs} \\
\text{Pour } \sigma_{\text{nom}} = M/c/I \text{ des profilés en I : } & \quad \sigma_p (\text{profilés en I}) = 1.1 \cdot \frac{S_e}{fs} \\
\text{Pour } \tau_{\text{nom}} = T/r/J \text{ des tubes minces : } & \quad \tau_p (\text{torse des tubes minces}) = \frac{\tau_e}{fs} \\
\text{Pour } \sigma_{\text{nom}} = M/r/I \text{ des tubes minces : } & \quad \sigma_p (\text{flexion des tubes minces}) = 1.27 \cdot \frac{S_e}{fs} \\
\text{Pour } \tau_{\text{nom}} = 16 \cdot T/\pi D^3, \text{ arbre plein : } & \quad \tau_p (\text{arbre plein}) = 1.33 \cdot \frac{\tau_e}{fs} \\
\text{Pour } \sigma_{\text{nom}} = 6 \cdot M/(bh^2), \text{ sections rectangulaires : } & \quad \sigma_p (\text{flexion section rectangulaire}) \leq 1.5 \cdot \frac{S_e}{fs} \\
\text{Pour } \sigma_{\text{nom}} = 32 \cdot M/(\pi D^3) : & \quad \sigma_p (\text{flexion circulaire plein}) \leq 1.7 \cdot \frac{S_e}{fs} \\
\end{align*}
\]

* Stress criteria for combined loads: Similarly to limit analysis, mostly used criterion is

\[
\left( \frac{\sigma_{\text{bending}}}{\sigma_{\text{allowed bending}}} \right)^2 + \left( \frac{\tau_{\text{torse}}}{\tau_{\text{allowed torsion}}} \right)^2 + \left( \frac{\tau_{\text{shear force}}}{\tau_{\text{allowed shear force}}} \right)^2 \leq 1
\]

Example: Using basic strength of materials, calculate stresses due to individual loads (bending, torsion and shear force) and, using the criterion for combined loads shown above, calculate the allowable load \( W \) at end B for a safety factor \( fs = 2 \) knowing that the tube is made of steel with yield stress \( S_e = 270 \text{ MPa} \).

Answer: \( W_{\text{allowed}} = 14 \, 340 \text{ N} \) (same answer as given by limit analysis)
Chapter 4. Design of welded joints (6 h class + 4 h hands on FEM software)

[3 hours]- Design of structural welded joints

Introduction to:
* Welds that need calculation;
* Weld treated as lines;
* Linear force on weld (f):
  \[ f = \lim_{\Delta L \to 0} \frac{\Delta F}{\Delta L} \]

* Practical formulas for calculating linear force f:
  \[ f_y = \frac{F_x}{L} \cdot T_y \quad f_x = \frac{F_y}{L} + T_x \]
  \[ f_z = \frac{M_x}{I_{G_y}} + \frac{M_y}{I_{G_x}} \]

Resultant linear force \[ \sqrt{f_y^2 + f_z^2 + f_x^2} \] treated as shear force

* Canadian Standard Association criterion for weld treated as lines:
  \[ \frac{f}{t_w} \leq 0.3 \cdot S_u \]

Type 1 example: \[ F = 15000 \text{ N} \]
(a) Calculate shear stress in weld at A by treating weld as a line.
Answer: 144 MPa
(b) Study this assembly by using a FEM software and determine:
(b.1) Maximum Von-Mises stress in weld. Answer: not converged
(b.2) Membrane shear stress across weld throat A. Ans.: 149 MPa

[3 hours]- Design of longitudinal welds of beams

* Linear force on longitudinal weld of beams due to shear for \( V \):
  \[ f = \frac{VA_1Y_1}{nL} \]

Type 2 example: poutre ABC de la figure ci-contre est fabriquée par l'assemblage d'un profilé W 410 x 74 (A = 9480 mm\(^4\), \( I_x = 273 \times 10^6 \) mm\(^4\), \( b = 180 \text{ mm} \), \( t_s = 16 \text{ mm} \), \( t_a = 9.7 \text{ mm} \), \( h = 413 \text{ mm} \)) à une plaque de 400 mm x 24 mm. Pour le chargement tel que montré, (1) déterminer la taille requise de la soudure d'angle continue et (2) la soudure intermittente si nécessaire. L'électrode utilisée est du type E480XX.

Rép. : (1) \( I_{total} = 501.2 \times 10^6 \) mm\(^4\), \( a_r = 2.05 \text{ mm} \),
(2) 7 mm-40 mm-130 mm
Chapter 5. Design of structural bolted joints (3 h class + 2 h hands on FEM software)

Criterion of combined loads for bolts:

\[ \frac{F_{cb}}{A_k S_b} + \left( \frac{F_{bh}}{A_b \tau_y} \right)^2 \leq 1 \text{ ou } r^3 = \left( \frac{F_{cb}}{A_k S_b} \right)^2 + \left( \frac{F_{bh}}{0.577 A_b S_b} \right)^2 \leq 1 \]

Fig. 5.1 - Trois chargements possibles d’un boulon / rivet

P5.3 Pour l’accouplement boulonné tout en acier dont la moitié gauche est telle que montée, déterminer la force de serrage initiale ainsi que le diamètre requis des boulons pour qu’il n’y ait pas de glissement aux surfaces de contact avec un facteur de sécurité de \( f_{\text{gliss.}} = 1.5 \). Les données connues sont : \( L_b = 400 \text{ mm} \), \( N = 8 \) boulons, \( B = 600 \text{ mm} \), \( V = 40 \text{ kN} \), \( M = 120 \text{ kNm} \), \( T = 48 \text{ kNm} \), \( \mu = 0.25 \) (coefficient de frottement). Les brides et boulons sont en acier et les contraintes permises des boulons sont \( S_{\text{pt}} = 175 \text{ MPa} \) en tension et \( S_{\text{ps}} = 70 \text{ MPa} \) en cisaillement.

Rép. : 213.69 kN ; 40.34 mm (Lb/db = 9.92 > 6 donc c = 0.9 est raisonnable)

Details will be presented and discussed on site

Chapter 6. Structural design against fatigue (6 h class + 4 h / FEM)

Details will be presented and discussed on site

Chapter 7. Practical advanced notions of bending and torsion (3 h class + 2 h / FEM)

Details will be presented and discussed on site